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ABSTRACT | Carlos Braumman

Title

Stochastic models of growth and extinction of populations

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Abstract

There are two main sources of stochasticity in the growth of a population, the demographic stochasticity (resulting from sampling variations in births and deaths, which have deterministic rates) and the environmental stochasticity (effect of random environmental variations on birth and death rates). The demographic stochasticity is usually studied through branching processes or birth and death processes. The environmental stochastic is conveniently studied using stochastic differential equations (SDE). We will present a general environmental stochasticity model with constant noise intensitys given by the SDE $dN(t)=g(N(t))N(t)dt+sN(t)dW(t)$, where $N=N(t)$ is the population size at time t, g(N) (where g is an arbitrary smooth function satisfying only some conditions dictated by biological considerations) is the average growth rate, and W(t) is the standard Wiener process (approximately describing the cumulative effect up to time t of environmental fluctuations on the growth rate). However, since we want to compare the effect of demographic stochasticity with the effect of environmental stochasticity, we will first study the special bentchmark case of density-independent (Malthusian) growth (g(N) constant). This SDE model will be compared, with respect to population extinction and local behavior, with the classical density-independent demograpghic stochasticity models, namely the Galton-Watson process (a particular type of discrete-time branching process) and the simple birth-and-death process (a continuous-time process). We will then study the population extinction properties of a general SDE model with density-dependence (and even allowing the noise intensity to be also a density-dependent function $s=s(N)$). There are no similar results available for general density-dependent branching processes or birth-and-death processes, so we speculate on the extinction properties for such models and give suggestions for their investigation. For SDE models, we will obtain the mean and variance of the extinction time for a general density-dependence function g and study their behavior for specific commonly used functions g.